1. **You are going to play 2 games of chess with an opponent whom you have never played against before (for the sake of this problem). Your opponent is equally likely to be a beginner, intermediate, or a master. Depending on   
   (a) What is your probability of winning the first game?**

**(b) Congratulations: you won the first game! Given this information, what is the probability that you will also win the second game**

**(c) Explain the distinction between assuming that the outcomes of the games are independent and assuming that they are conditionally independent given the opponent’s skill level. Which of these assumptions seems more reasonable, and why?**

**Ans-** (a) Assuming that your opponent is equally likely to be a beginner, intermediate, or a master, and if we assume that your chance of winning against a beginner is 90%, against an intermediate player is 50%, and against a master player is 10%, then the probability of winning the first game can be calculated as follows:

P(win) = P(win against beginner) × P(opponent is beginner) + P(win against intermediate) × P(opponent is intermediate) + P(win against master) × P(opponent is master)

P(win) = (0.9 × 1/3) + (0.5 × 1/3) + (0.1 × 1/3)

P(win) = 0.5

Therefore, the probability of winning the first game is 0.5 or 50%.

(b) Given that you won the first game, the probability of winning the second game can be calculated using Bayes' theorem:

P(win second game | win first game) = P(win first game and win second game) / P(win first game)

The probability of winning both games depends on the opponent's skill level. If the opponent is a beginner, then the probability of winning both games is 0.9 × 0.9 = 0.81. If the opponent is an intermediate player, then the probability of winning both games is 0.5 × 0.5 = 0.25. If the opponent is a master, then the probability of winning both games is 0.1 × 0.1 = 0.01.

Using the law of total probability, the probability of winning the first game is:

P(win first game) = P(win first game | opponent is beginner) × P(opponent is beginner) + P(win first game | opponent is intermediate) × P(opponent is intermediate) + P(win first game | opponent is master) × P(opponent is master)

P(win first game) = (0.9 × 1/3) + (0.5 × 1/3) + (0.1 × 1/3)

P(win first game) = 0.5

Therefore,

P(win second game | win first game) = (0.81 × 1/3) + (0.25 × 1/3) + (0.01 × 1/3) / 0.5 = 0.3567

Thus, given that you won the first game, the probability of winning the second game is 0.3567 or approximately 36%.

(c) Assuming that the outcomes of the games are independent means that the outcome of the first game does not affect the outcome of the second game. In other words, the probability of winning the second game is the same, regardless of whether you won or lost the first game.

Assuming that the outcomes are conditionally independent given the opponent's skill level means that the outcome of the first game can affect the outcome of the second game, but only through the opponent's skill level. For example, if you win the first game against a master player, then it is more likely that you will win the second game against the same opponent because you have learned more about their playing style.

The assumption that the outcomes are conditionally independent given the opponent's skill level seems more reasonable because the outcome of the first game can provide valuable information about the opponent's skill level, which can affect the outcome of the second game. However, it is also important to consider other factors that could affect the outcomes of the games, such as fatigue, concentration, and psychological factors